

Let's work an example of §20.2(4), monads on a poset-considered-as-a-category,  $\mathcal{X}$ .

The existence of the monad's functor  $T : \mathcal{X} \rightarrow \mathcal{X}$  assures us that  $X \leq Y$  implies  $TX \leq TY$ ;  $\eta_X$  that  $X \leq TX$ , and  $\mu_X$  that  $TTX \leq TX$ .

It is important to note that because  $Id_{\mathcal{X}}$  is always a monad (with suitable choices for  $\eta_X$  and  $\mu_X$ ), merely requiring that  $T$  be a monad on  $\mathcal{X}$  is not enough to exclude the identity endofunctor. But let's reason directly and see what we get.

We can combine  $T$  and  $\eta_X$  to observe that  $T(\eta_X) : TX \leq TTX$ . Further combining  $\mu_X$ , we see that  $\forall X. TX \simeq TTX$ : posets have at most one arrow from one object to another, so  $TX \leq TTX$  and  $TTX \leq TX$  must compose to  $TX \leq TX$  and  $TTX \leq TTX$ , the identity arrows.

If  $X$  is maximal (i.e.,  $\nexists Y. X < Y$ ), then  $\eta_X : X \leq TX$  implies that  $TX = X$ , as  $\eta_X = id_X$  is the only choice.

Elsewhere,  $X = TX = TTX$  is entirely acceptable ( $\eta_X = \mu_X = id_X$ ), but not required.